Extending Coordinated Universal Time to Dates Before 1972[*]

John Sauter[†]

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Abstract

Using ancient observations of the Sun and Moon, construct a time scale using the modern definition of Coordinated Universal Time to cover the past 4000 years. Use that time scale to construct a table of leap seconds.

Keywords: Coordinated Universal Time; UTC; proleptic UTC; Gregorian calendar; proleptic Gregorian calendar; leap seconds; delta T; ΔT; proleptic UTC with leap seconds.

URL: https://www.systemeyescomputerstore.com/leap_seconds/proleptic_UTC.pdf

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[†] System Eyes Computer Store, 20A Northwest Blvd. Ste 345, Nashua, NH 03063-4066, e-mail: John_Sauter@systemeyescomputerstore.com, telephone: (603) 424-1188

1 Civil Timekeeping

Coordinated Universal Time (UTC) is the basis of civil time in the world today. Local time is defined as an offset from UTC. In places which observe daylight saving time, the offset changes when daylight saving time begins and ends.

UTC is based on the SI second, which is a fixed interval of time, independent of the motions of the Earth. However, for civil purposes timekeeping has always been based on the days and the seasons. The duration of Earth's day is not fixed: it has small random variations, and is gradually increasing as the rotation of the Earth slows. In addition, the year is not an integral number of days in length. We use a complex system of timekeeping to reconcile a fixed time interval with the changing length of the day, and the days with the seasons.

2 The Gregorian Calendar

To fit days into years we use the Gregorian calendar, which provides 12 months of 28 to 31 days each, for a total of 365 or 366 days in a year. The 366-day years add a day after February 28 called February 29. The rule for adding February 29 is that every year evenly divisible by 4 shall have a February 29, unless the year is also evenly divisible by 100. However, if the year is also evenly divisible by 400 it shall have a February 29.

3 Matching SI Seconds to the Rotation of the Earth

To fit the SI second into days, UTC starts a new day every 86 399, 86 400 or 86 401 seconds. Most days are ordinary days with 86 400 seconds. The last day of a month can be an extraordinary day, with 86 399 or 86 401 seconds. In an extraordinary day with 86 399 seconds, the last second of the day is called 23:59:58. In an ordinary day with 86 400 seconds, the last second of the day is called 23:59:59. In an extraordinary day with 86 401 seconds, the last second of the day is called 23:59:60.

The extraordinary days are scheduled by the International Earth Rotation and Reference Systems Service (IERS)[1] based on observations of the Earth's rotation. A long day is used when UTC is counting faster than the Earth is rotating, and a short day is used when UTC is counting slower than the Earth is rotating.

4 A Brief History of Timekeeping

The Gregorian calendar was adopted starting in 1582 because the seasons had slowly changed their dates under the previous calendar. The changeover included

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[1] https://www.iers.org/IERS/EN/Home/home_node.html

dropping 10 days from the calendar to correct the accumulated error. Since the adoption of the Gregorian calendar, the years 1700, 1800 and 1900 have not included February 29, but the years 1600 and 2000 have.

The IERS began scheduling extraordinary days in 1972 so that civil time could use the SI second rather than a variable-length second. As of the time of this writing, in April of 2025, there have been 27 days with 86 401 seconds and 0 days with 86 399 seconds.

Before 1972 there was a time scale also called UTC, but which used a different method of matching SI seconds to the Earth's rotation. In this paper the term "UTC" refers to the UTC used since 1972.

5 Proleptic Gregorian Calendar

For some purposes, it is useful to extend the rules of the Gregorian calendar to dates before 1582. For example, when describing Mayan culture for a general audience, the mythical creation date of 13 b'ak'tuns, 0 k'atuns, 0 tuns, 0 winals, 0 kins (4 Ajaw, 8 Kumk'u) is translated as August 11, 3114 BCE. Mayan scholars generally translate Mesoamerican Long Count calendar dates into Proleptic Gregorian dates rather than the Julian calendar dates used for civil timekeeping in Europe at the time of the Mayan civilization because, to a modern audience, the proleptic Gregorian date conveys the season of the year more accurately.

6 Proleptic UTC

If we are to specify ancient times with a greater precision than a day, it will also be useful for some purposes to extend the rules of UTC to times before 1972. To do this, we need two things: a time scale and the rotation rate of the Earth in the past.

6.1 Time Scale

For a time scale to be useful for measuring events, it must be based on some physical property that can be measured. UTC is based on the SI second, which is defined as the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom when it is at rest with respect to the Earth, at a temperature of 0 Kelvin, and located at mean sea level. These measurements are made continuously by national laboratories around the world. The data is sent to the International Bureau of Weights and Measures, which combines the information and disseminates International Atomic Time, upon which UTC is based.

A time scale used for scientific measurements must be rigorously defined and robustly measured, as International Atomic Time is. If we try to extend this definition backward into ancient times, we run into the problem that there were no atomic clocks before 1955, and therefore nobody was measuring the SI second as it is presently defined.

However, when describing ancient events, we do not need to actually measure time, we can instead imagine that we are measuring it. For example, we say that the Big Bang that marked the start of the Universe took place 13.787 billion years ago. In this context, “years” does not refer to the number of times that the Earth has gone around the Sun since the Big Bang, because the Earth has only been in existence for the last 4.54 billion years. Instead, we take the present duration of a year and project it back in time, thus imagining that we are measuring time using the year as it presently exists.

Similarly, we can imagine that atomic clocks have been measuring time for the last 4000 years, and use that as our time scale. This imaginary time scale does not have the precision of International Atomic Time, since there were not, in fact, atomic clocks 4000 years ago, but it is sufficient for our limited purpose.

6.2 Rotation Rate of the Earth

The Earth revolves around the Sun at a constant rate, so the schedule of 366-day years is predictable. The Earth’s rotation rate, on the other hand, is unpredictable, and must be observed. That observation is described as ΔT, which is the difference between Terrestrial Time[2]and UTC. Using historical records of astronomical observations, F. R. Stephenson, L. V. Morrison, C. Y. Hohenkerk and M. Zawilski have estimated the values of ΔT from the year −2000 to 2500[Stephenson et al., 2016]. The table below is based on the data calculated in that paper, presented at http://astro.ukho.gov.uk/nao/lvm/, as updated in February 2021[Morrison et al., 2021]. Where the data presented two different values of ΔT for the same date, I have averaged the values. In the table below I have presented ΔT as seconds, omitted the estimated error, and added a column showing the Julian Day Number for each date. The Julian Day Number is the count of days since noon on November 24, −4713, so using it to measure time avoids the complexity of the Gregorian calendar. Because the Julian Day starts at noon, adding 0.5 means the midnight which starts the next day.

Table 1: Values of ΔT from 1 Jan –2000 to 1 Jan 2500

Date	ΔT	Julian Day
1 Jan –2000	46 080.0	990 574.5
1 Jan –1900	43 560.0	1 027 099.5
1 Jan –1800	41 040.0	1 063 623.5
1 Jan –1700	38 880.0	1 100 147.5
1 Jan –1600	36 720.0	1 136 671.5
1 Jan –1500	34 920.0	1 173 196.5
1 Jan –1400	32 760.0	1 209 720.5

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[2] Terrestrial Time differs from International Atomic Time by 32.184 seconds.

Table 1: Values of ΔT from 1 Jan –2000 to 1 Jan 2500

Date	ΔT	Julian Day
1 Jan –1300	30 960.0	1 246 244.5
1 Jan –1200	29 160.0	1 282 768.5
1 Jan –1100	27 360.0	1 319 293.5
1 Jan –1000	25 560.0	1 355 817.5
1 Jan –900	23 760.0	1 392 341.5
1 Jan –800	21 960.0	1 428 865.5
1 Jan –720	20 370.0	1 458 085.5
1 Jan –700	20 050.0	1 465 390.5
1 Jan –600	18 470.0	1 501 914.5
1 Jan –500	16 940.0	1 538 438.5
1 Jan –400	15 470.0	1 574 962.5
1 Jan –300	14 080.0	1 611 487.5
1 Jan –200	12 770.0	1 648 011.5
1 Jan –100	11 560.0	1 684 535.5
1 Jan 0	10 440.0	1 721 059.5
1 Jan 100	9410.0	1 757 584.5
1 Jan 200	8420.0	1 794 108.5
1 Jan 300	7480.0	1 830 632.5
1 Jan 400	6540.0	1 867 156.5
1 Jan 500	5590.0	1 903 681.5
1 Jan 600	4650.0	1 940 205.5
1 Jan 700	3760.0	1 976 729.5
1 Jan 800	2940.0	2 013 253.5
1 Jan 900	2230.0	2 049 778.5
1 Jan 1000	1650.0	2 086 302.5
1 Jan 1100	1220.0	2 122 826.5
1 Jan 1200	910.0	2 159 350.5
1 Jan 1300	680.0	2 195 875.5
1 Jan 1400	480.0	2 232 399.5
1 Jan 1500	290.0	2 268 923.5
1 Jan 1600	109.5	2 305 447.5

Table 1: Values of ΔT from 1 Jan –2000 to 1 Jan 2500

Date	ΔT	Julian Day
1 Jan 1610	94.0	2 309 100.5
1 Jan 1620	80.0	2 312 752.5
1 Jan 1630	66.0	2 316 405.5
1 Jan 1640	54.0	2 320 057.5
1 Jan 1650	44.0	2 323 710.5
1 Jan 1660	35.0	2 327 362.5
1 Jan 1670	28.0	2 331 015.5
1 Jan 1680	22.0	2 334 667.5
1 Jan 1690	17.0	2 338 320.5
1 Jan 1700	14.0	2 341 972.5
1 Jan 1710	12.0	2 345 624.5
1 Jan 1720	12.0	2 349 276.5
1 Jan 1730	13.0	2 352 929.5
1 Jan 1740	15.0	2 356 581.5
1 Jan 1750	17.0	2 360 234.5
1 Jan 1760	19.0	2 363 886.5
1 Jan 1770	21.0	2 367 539.5
1 Jan 1780	21.0	2 371 191.5
1 Jan 1790	21.0	2 374 844.5
1 Jan 1800	18.2	2 378 496.5
1 Jan 1801	18.0	2 378 861.5
1 Jan 1802	17.6	2 379 226.5
1 Jan 1803	17.3	2 379 591.5
1 Jan 1804	16.9	2 379 956.5
1 Jan 1805	16.6	2 380 322.5
1 Jan 1806	16.3	2 380 687.5
1 Jan 1807	16.0	2 381 052.5
1 Jan 1808	15.8	2 381 417.5
1 Jan 1809	15.7	2 381 783.5
1 Jan 1810	15.85	2 382 148.5
1 Jan 1811	15.7	2 382 513.5

Table 1: Values of ΔT from 1 Jan –2000 to 1 Jan 2500

Date	ΔT	Julian Day
1 Jan 1812	15.8	2 382 878.5
1 Jan 1813	16.0	2 383 244.5
1 Jan 1814	16.2	2 383 609.5
1 Jan 1815	16.4	2 383 974.5
1 Jan 1816	16.5	2 384 339.5
1 Jan 1817	16.7	2 384 705.5
1 Jan 1818	16.7	2 385 070.5
1 Jan 1819	16.7	2 385 435.5
1 Jan 1820	16.5	2 385 800.5
1 Jan 1821	16.2	2 386 166.5
1 Jan 1822	15.8	2 386 531.5
1 Jan 1823	15.3	2 386 896.5
1 Jan 1824	14.8	2 387 261.5
1 Jan 1825	14.1	2 387 627.5
1 Jan 1826	13.5	2 387 992.5
1 Jan 1827	12.8	2 388 357.5
1 Jan 1828	12.1	2 388 722.5
1 Jan 1829	11.4	2 389 088.5
1 Jan 1830	10.8	2 389 453.5
1 Jan 1831	10.2	2 389 818.5
1 Jan 1832	9.7	2 390 183.5
1 Jan 1833	9.3	2 390 549.5
1 Jan 1834	8.9	2 390 914.5
1 Jan 1835	8.5	2 391 279.5
1 Jan 1836	8.2	2 391 644.5
1 Jan 1837	8.0	2 392 010.5
1 Jan 1838	7.8	2 392 375.5
1 Jan 1839	7.7	2 392 740.5
1 Jan 1840	7.6	2 393 105.5
1 Jan 1841	7.6	2 393 471.5
1 Jan 1842	7.7	2 393 836.5

Table 1: Values of ΔT from 1 Jan –2000 to 1 Jan 2500

Date	ΔT	Julian Day
1 Jan 1843	7.7	2 394 201.5
1 Jan 1844	7.9	2 394 566.5
1 Jan 1845	8.0	2 394 932.5
1 Jan 1846	8.2	2 395 297.5
1 Jan 1847	8.5	2 395 662.5
1 Jan 1848	8.7	2 396 027.5
1 Jan 1849	9.0	2 396 393.5
1 Jan 1850	9.3	2 396 758.5
1 Jan 1851	9.67	2 397 123.5
1 Jan 1852	9.98	2 397 488.5
1 Jan 1853	10.23	2 397 854.5
1 Jan 1854	10.37	2 398 219.5
1 Jan 1855	10.36	2 398 584.5
1 Jan 1856	10.18	2 398 949.5
1 Jan 1857	9.88	2 399 315.5
1 Jan 1858	9.54	2 399 680.5
1 Jan 1859	9.24	2 400 045.5
1 Jan 1860	9.04	2 400 410.5
1 Jan 1861	8.99	2 400 776.5
1 Jan 1862	9.01	2 401 141.5
1 Jan 1863	8.97	2 401 506.5
1 Jan 1864	8.76	2 401 871.5
1 Jan 1865	8.25	2 402 237.5
1 Jan 1866	7.38	2 402 602.5
1 Jan 1867	6.22	2 402 967.5
1 Jan 1868	4.92	2 403 332.5
1 Jan 1869	3.59	2 403 698.5
1 Jan 1870	2.37	2 404 063.5
1 Jan 1871	1.36	2 404 428.5
1 Jan 1872	0.56	2 404 793.5
1 Jan 1873	−0.1	2 405 159.5

Table 1: Values of ΔT from 1 Jan –2000 to 1 Jan 2500

Date	ΔT	Julian Day
1 Jan 1874	−0.65	2 405 524.5
1 Jan 1875	−1.13	2 405 889.5
1 Jan 1876	−1.58	2 406 254.5
1 Jan 1877	−2.01	2 406 620.5
1 Jan 1878	−2.43	2 406 985.5
1 Jan 1879	−2.83	2 407 350.5
1 Jan 1880	−3.21	2 407 715.5
1 Jan 1881	−3.58	2 408 081.5
1 Jan 1882	−3.91	2 408 446.5
1 Jan 1883	−4.17	2 408 811.5
1 Jan 1884	−4.34	2 409 176.5
1 Jan 1885	−4.39	2 409 542.5
1 Jan 1886	−4.31	2 409 907.5
1 Jan 1887	−4.14	2 410 272.5
1 Jan 1888	−3.97	2 410 637.5
1 Jan 1889	−3.86	2 411 003.5
1 Jan 1890	−3.88	2 411 368.5
1 Jan 1891	−4.07	2 411 733.5
1 Jan 1892	−4.37	2 412 098.5
1 Jan 1893	−4.69	2 412 464.5
1 Jan 1894	−4.93	2 412 829.5
1 Jan 1895	−5.02	2 413 194.5
1 Jan 1896	−4.87	2 413 559.5
1 Jan 1897	−4.48	2 413 925.5
1 Jan 1898	−3.86	2 414 290.5
1 Jan 1899	−3.02	2 414 655.5
1 Jan 1900	−1.98	2 415 020.5
1 Jan 1901	−0.75	2 415 385.5
1 Jan 1902	0.62	2 415 750.5
1 Jan 1903	2.06	2 416 115.5
1 Jan 1904	3.51	2 416 480.5

Table 1: Values of ΔT from 1 Jan –2000 to 1 Jan 2500

Date	ΔT	Julian Day
1 Jan 1905	4.92	2 416 846.5
1 Jan 1906	6.24	2 417 211.5
1 Jan 1907	7.49	2 417 576.5
1 Jan 1908	8.7	2 417 941.5
1 Jan 1909	9.9	2 418 307.5
1 Jan 1910	11.14	2 418 672.5
1 Jan 1911	12.43	2 419 037.5
1 Jan 1912	13.75	2 419 402.5
1 Jan 1913	15.06	2 419 768.5
1 Jan 1914	16.32	2 420 133.5
1 Jan 1915	17.48	2 420 498.5
1 Jan 1916	18.52	2 420 863.5
1 Jan 1917	19.44	2 421 229.5
1 Jan 1918	20.25	2 421 594.5
1 Jan 1919	20.98	2 421 959.5
1 Jan 1920	21.62	2 422 324.5
1 Jan 1921	22.19	2 422 690.5
1 Jan 1922	22.69	2 423 055.5
1 Jan 1923	23.12	2 423 420.5
1 Jan 1924	23.49	2 423 785.5
1 Jan 1925	23.79	2 424 151.5
1 Jan 1926	24.02	2 424 516.5
1 Jan 1927	24.2	2 424 881.5
1 Jan 1928	24.32	2 425 246.5
1 Jan 1929	24.39	2 425 612.5
1 Jan 1930	24.42	2 425 977.5
1 Jan 1931	24.41	2 426 342.5
1 Jan 1932	24.38	2 426 707.5
1 Jan 1933	24.32	2 427 073.5
1 Jan 1934	24.25	2 427 438.5
1 Jan 1935	24.16	2 427 803.5

Table 1: Values of ΔT from 1 Jan –2000 to 1 Jan 2500

Date	ΔT	Julian Day
1 Jan 1936	24.08	2 428 168.5
1 Jan 1937	24.04	2 428 534.5
1 Jan 1938	24.06	2 428 899.5
1 Jan 1939	24.17	2 429 264.5
1 Jan 1940	24.43	2 429 629.5
1 Jan 1941	24.83	2 429 995.5
1 Jan 1942	25.35	2 430 360.5
1 Jan 1943	25.92	2 430 725.5
1 Jan 1944	26.51	2 431 090.5
1 Jan 1945	27.05	2 431 456.5
1 Jan 1946	27.51	2 431 821.5
1 Jan 1947	27.89	2 432 186.5
1 Jan 1948	28.24	2 432 551.5
1 Jan 1949	28.58	2 432 917.5
1 Jan 1950	28.93	2 433 282.5
1 Jul 1950	29.12	2 433 463.5
1 Jan 1951	29.32	2 433 647.5
1 Jul 1951	29.52	2 433 828.5
1 Jan 1952	29.7	2 434 012.5
1 Jul 1952	29.86	2 434 194.5
1 Jan 1953	30.0	2 434 378.5
1 Jul 1953	30.11	2 434 559.5
1 Jan 1954	30.2	2 434 743.5
1 Jul 1954	30.3	2 434 924.5
1 Jan 1955	30.41	2 435 108.5
1 Jul 1955	30.56	2 435 289.5
1 Jan 1956	30.76	2 435 473.5
1 Jul 1956	31.03	2 435 655.5
1 Jan 1957	31.34	2 435 839.5
1 Jul 1957	31.68	2 436 020.5
1 Jan 1958	32.03	2 436 204.5

Table 1: Values of ΔT from 1 Jan –2000 to 1 Jan 2500

Date	ΔT	Julian Day
1 Jul 1958	32.36	2 436 385.5
1 Jan 1959	32.65	2 436 569.5
1 Jul 1959	32.89	2 436 750.5
1 Jan 1960	33.07	2 436 934.5
1 Jul 1960	33.23	2 437 116.5
1 Jan 1961	33.36	2 437 300.5
1 Jul 1961	33.49	2 437 481.5
1 Jan 1962	33.62	2 437 665.5
1 Jul 1962	33.78	2 437 846.5
1 Jan 1963	33.96	2 438 030.5
1 Jul 1963	34.18	2 438 211.5
1 Jan 1964	34.44	2 438 395.5
1 Jul 1964	34.74	2 438 577.5
1 Jan 1965	35.09	2 438 761.5
1 Jul 1965	35.5	2 438 942.5
1 Jan 1966	35.95	2 439 126.5
1 Jul 1966	36.43	2 439 307.5
1 Jan 1967	36.93	2 439 491.5
1 Jul 1967	37.45	2 439 672.5
1 Jan 1968	37.96	2 439 856.5
1 Jul 1968	38.46	2 440 038.5
1 Jan 1969	38.95	2 440 222.5
1 Jul 1969	39.44	2 440 403.5
1 Jan 1970	39.93	2 440 587.5
1 Jul 1970	40.43	2 440 768.5
1 Jan 1971	40.95	2 440 952.5
1 Jul 1971	41.49	2 441 133.5
1 Jan 1972	42.04	2 441 317.5
1 Jul 1972	42.59	2 441 499.5
1 Jan 1973	43.15	2 441 683.5
1 Jul 1973	43.7	2 441 864.5

Table 1: Values of ΔT from 1 Jan –2000 to 1 Jan 2500

Date	ΔT	Julian Day
1 Jan 1974	44.24	2 442 048.5
1 Jul 1974	44.77	2 442 229.5
1 Jan 1975	45.28	2 442 413.5
1 Jul 1975	45.78	2 442 594.5
1 Jan 1976	46.28	2 442 778.5
1 Jul 1976	46.78	2 442 960.5
1 Jan 1977	47.29	2 443 144.5
1 Jul 1977	47.81	2 443 325.5
1 Jan 1978	48.33	2 443 509.5
1 Jul 1978	48.85	2 443 690.5
1 Jan 1979	49.37	2 443 874.5
1 Jul 1979	49.87	2 444 055.5
1 Jan 1980	50.36	2 444 239.5
1 Jul 1980	50.83	2 444 421.5
1 Jan 1981	51.28	2 444 605.5
1 Jul 1981	51.71	2 444 786.5
1 Jan 1982	52.13	2 444 970.5
1 Jul 1982	52.54	2 445 151.5
1 Jan 1983	52.94	2 445 335.5
1 Jul 1983	53.32	2 445 516.5
1 Jan 1984	53.7	2 445 700.5
1 Jul 1984	54.06	2 445 882.5
1 Jan 1985	54.39	2 446 066.5
1 Jul 1985	54.7	2 446 247.5
1 Jan 1986	54.98	2 446 431.5
1 Jul 1986	55.23	2 446 612.5
1 Jan 1987	55.46	2 446 796.5
1 Jul 1987	55.67	2 446 977.5
1 Jan 1988	55.89	2 447 161.5
1 Jul 1988	56.12	2 447 343.5
1 Jan 1989	56.37	2 447 527.5

Table 1: Values of ΔT from 1 Jan –2000 to 1 Jan 2500

Date	ΔT	Julian Day
1 Jul 1989	56.67	2 447 708.5
1 Jan 1990	56.99	2 447 892.5
1 Jul 1990	57.34	2 448 073.5
1 Jan 1991	57.7	2 448 257.5
1 Jul 1991	58.08	2 448 438.5
1 Jan 1992	58.45	2 448 622.5
1 Jul 1992	58.82	2 448 804.5
1 Jan 1993	59.19	2 448 988.5
1 Jul 1993	59.55	2 449 169.5
1 Jan 1994	59.92	2 449 353.5
1 Jul 1994	60.3	2 449 534.5
1 Jan 1995	60.68	2 449 718.5
1 Jul 1995	61.07	2 449 899.5
1 Jan 1996	61.46	2 450 083.5
1 Jul 1996	61.85	2 450 265.5
1 Jan 1997	62.23	2 450 449.5
1 Jul 1997	62.58	2 450 630.5
1 Jan 1998	62.9	2 450 814.5
1 Jul 1998	63.18	2 450 995.5
1 Jan 1999	63.42	2 451 179.5
1 Jul 1999	63.63	2 451 360.5
1 Jan 2000	63.81	2 451 544.5
1 Jul 2000	63.96	2 451 726.5
1 Jan 2001	64.08	2 451 910.5
1 Jul 2001	64.18	2 452 091.5
1 Jan 2002	64.27	2 452 275.5
1 Jul 2002	64.34	2 452 456.5
1 Jan 2003	64.41	2 452 640.5
1 Jul 2003	64.48	2 452 821.5
1 Jan 2004	64.55	2 453 005.5
1 Jul 2004	64.64	2 453 187.5

Table 1: Values of ΔT from 1 Jan –2000 to 1 Jan 2500

Date	ΔT	Julian Day
1 Jan 2005	64.73	2 453 371.5
1 Jul 2005	64.83	2 453 552.5
1 Jan 2006	64.95	2 453 736.5
1 Jul 2006	65.07	2 453 917.5
1 Jan 2007	65.2	2 454 101.5
1 Jul 2007	65.33	2 454 282.5
1 Jan 2008	65.48	2 454 466.5
1 Jul 2008	65.62	2 454 648.5
1 Jan 2009	65.77	2 454 832.5
1 Jul 2009	65.92	2 455 013.5
1 Jan 2010	66.06	2 455 197.5
1 Jul 2010	66.2	2 455 378.5
1 Jan 2011	66.33	2 455 562.5
1 Jul 2011	66.47	2 455 743.5
1 Jan 2012	66.61	2 455 927.5
1 Jul 2012	66.76	2 456 109.5
1 Jan 2013	66.92	2 456 293.5
1 Jul 2013	67.09	2 456 474.5
1 Jan 2014	67.28	2 456 658.5
1 Jul 2014	67.48	2 456 839.5
1 Jan 2015	67.69	2 457 023.5
1 Jul 2015	67.9	2 457 204.5
1 Jan 2016	68.11	2 457 388.5
1 Jul 2016	68.32	2 457 570.5
1 Jan 2017	68.53	2 457 754.5
1 Jul 2017	68.73	2 457 935.5
1 Jan 2018	68.92	2 458 119.5
1 Jul 2018	69.09	2 458 300.5
1 Jan 2019	69.24	2 458 484.5
1 Jan 2030	67.0	2 462 502.5
1 Jan 2040	68.0	2 466 154.5

Table 1: Values of ΔT from 1 Jan –2000 to 1 Jan 2500

Date	ΔT	Julian Day
1 Jan 2050	70.0	2 469 807.5
1 Jan 2100	80.0	2 488 069.5
1 Jan 2200	160.0	2 524 593.5
1 Jan 2300	330.0	2 561 117.5
1 Jan 2400	610.0	2 597 641.5
1 Jan 2500	1000.0	2 634 166.5

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7 Schedule of Extraordinary Days

Using this information we can schedule extraordinary days as though UTC were being used prior to 1972. In 2006 Tony Finch created a proleptic UTC back to 1958 using US Naval Observatory tables[3] so I will follow his lead for those dates.

To schedule extraordinary days, we establish an anchor point and march forward through time, observing the deviation of ΔT from the value at the anchor point. Before the deviation reaches 0.9 seconds, we must designate an extraordinary day. If the interval includes a June 30 or December 31, we will designate that day. If it does not, the next dates to choose will be March 31 and September 30, and if they are not available we will designate the last day of any month. The procedure is based on the formal definition of UTC..[4] However, if the value of ΔT changes so much that two extraordinary days are needed in the same month, we designate the 15th and the last day of the month. Details of this process are in file `read_delta_t.py`, which is embedded in this PDF file, as described in section 12.

At the end of each extraordinary day, the difference between UTC and International Atomic Time changes by one second. That difference is called DTAI, and it is defined to be 0 at the beginning of January 1, 1958.

The result of the scheduling process is too large for this paper, but here are the extraordinary days from 1700 to 2100. The complete table is embedded in this PDF under the name `extraordinary_days.dat`, as described in section 12. Extraordinary days before 1958 are computed based on the estimates of ΔT, as described above. From 1958 through 1971 the dates are from Tony Finch. From 1972 to the present they are the official days scheduled by the International Earth Rotation and Reference Systems Service (IERS).[5] The schedule for the

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[3] https://fanf.livejournal.com/69586.html

[4] The definition of UTC is maintained by the International Telecommunications Union as their document ITU-R TF.460-6, which you can see at https://www.itu.int/dms_pubrec/itu-r/rec/tf/R-REC-TF.460-6-200202-I!!PDF-E.pdf

[5] https://www.iers.org/IERS/EN/DataProducts/EarthOrientationData/eop.html

future is based on projections from the IERS and astronomical projections of ΔT.

Table 2: Extraordinary days from 1 Jan 1700 to 2 Jan 2100

Julian Day Number	length in seconds	DTAI	Day Month Year
2 343 066.5	86 399	−19	# 31 Dec 1702
2 344 892.5	86 399	−20	# 31 Dec 1707
2 351 467.5	86 401	−19	# 31 Dec 1725
2 354 024.5	86 401	−18	# 31 Dec 1732
2 355 850.5	86 401	−17	# 31 Dec 1737
2 357 676.5	86 401	−16	# 31 Dec 1742
2 359 502.5	86 401	−15	# 31 Dec 1747
2 361 329.5	86 401	−14	# 31 Dec 1752
2 363 155.5	86 401	−13	# 31 Dec 1757
2 364 981.5	86 401	−12	# 31 Dec 1762
2 366 807.5	86 401	−11	# 31 Dec 1767
2 375 573.5	86 399	−12	# 31 Dec 1791
2 376 850.5	86 399	−13	# 30 Jun 1795
2 378 311.5	86 399	−14	# 30 Jun 1799
2 379 406.5	86 399	−15	# 30 Jun 1802
2 380 502.5	86 399	−16	# 30 Jun 1805
2 386 895.5	86 399	−17	# 31 Dec 1822
2 387 442.5	86 399	−18	# 30 Jun 1824
2 387 991.5	86 399	−19	# 31 Dec 1825
2 388 537.5	86 399	−20	# 30 Jun 1827
2 389 087.5	86 399	−21	# 31 Dec 1828
2 389 633.5	86 399	−22	# 30 Jun 1830
2 390 548.5	86 399	−23	# 31 Dec 1832
2 391 459.5	86 399	−24	# 30 Jun 1835
2 395 842.5	86 401	−23	# 30 Jun 1847
2 397 122.5	86 401	−22	# 31 Dec 1850
2 399 860.5	86 399	−23	# 30 Jun 1858
2 402 052.5	86 399	−24	# 30 Jun 1864
2 402 601.5	86 399	−25	# 31 Dec 1865

Table 2: Extraordinary days from 1 Jan 1700 to 2 Jan 2100

Julian Day Number	length in seconds	DTAI	Day Month Year
2 402 966.5	86 399	−26	# 31 Dec 1866
2 403 147.5	86 399	−27	# 30 Jun 1867
2 403 513.5	86 399	−28	# 30 Jun 1868
2 403 697.5	86 399	−29	# 31 Dec 1868
2 404 062.5	86 399	−30	# 31 Dec 1869
2 404 427.5	86 399	−31	# 31 Dec 1870
2 404 792.5	86 399	−32	# 31 Dec 1871
2 405 523.5	86 399	−33	# 31 Dec 1873
2 406 253.5	86 399	−34	# 31 Dec 1875
2 407 165.5	86 399	−35	# 30 Jun 1878
2 408 080.5	86 399	−36	# 31 Dec 1880
2 412 279.5	86 399	−37	# 30 Jun 1892
2 413 924.5	86 401	−36	# 31 Dec 1896
2 414 470.5	86 401	−35	# 30 Jun 1898
2 414 835.5	86 401	−34	# 30 Jun 1899
2 415 200.5	86 401	−33	# 30 Jun 1900
2 415 384.5	86 401	−32	# 31 Dec 1900
2 415 749.5	86 401	−31	# 31 Dec 1901
2 415 930.5	86 401	−30	# 30 Jun 1902
2 416 295.5	86 401	−29	# 30 Jun 1903
2 416 479.5	86 401	−28	# 31 Dec 1903
2 416 661.5	86 401	−27	# 30 Jun 1904
2 417 026.5	86 401	−26	# 30 Jun 1905
2 417 391.5	86 401	−25	# 30 Jun 1906
2 417 575.5	86 401	−24	# 31 Dec 1906
2 417 940.5	86 401	−23	# 31 Dec 1907
2 418 306.5	86 401	−22	# 31 Dec 1908
2 418 487.5	86 401	−21	# 30 Jun 1909
2 418 852.5	86 401	−20	# 30 Jun 1910
2 419 036.5	86 401	−19	# 31 Dec 1910
2 419 401.5	86 401	−18	# 31 Dec 1911

Table 2: Extraordinary days from 1 Jan 1700 to 2 Jan 2100

Julian Day Number	length in seconds	DTAI	Day Month Year
2 419 583.5	86 401	−17	# 30 Jun 1912
2 419 948.5	86 401	−16	# 30 Jun 1913
2 420 132.5	86 401	−15	# 31 Dec 1913
2 420 497.5	86 401	−14	# 31 Dec 1914
2 420 862.5	86 401	−13	# 31 Dec 1915
2 421 228.5	86 401	−12	# 31 Dec 1916
2 421 774.5	86 401	−11	# 30 Jun 1918
2 422 323.5	86 401	−10	# 31 Dec 1919
2 423 054.5	86 401	−9	# 31 Dec 1921
2 423 966.5	86 401	−8	# 30 Jun 1924
2 429 810.5	86 401	−7	# 30 Jun 1940
2 430 540.5	86 401	−6	# 30 Jun 1942
2 431 089.5	86 401	−5	# 31 Dec 1943
2 431 820.5	86 401	−4	# 31 Dec 1945
2 432 916.5	86 401	−3	# 31 Dec 1948
2 433 827.5	86 401	−2	# 30 Jun 1951
2 435 288.5	86 401	−1	# 30 Jun 1955
2 436 019.5	86 401	0	# 30 Jun 1957
2 436 749.5	86 401	1	# 30 Jun 1959
2 437 480.5	86 401	2	# 30 Jun 1961
2 438 210.5	86 401	3	# 30 Jun 1963
2 438 760.5	86 401	4	# 31 Dec 1964
2 439 306.5	86 401	5	# 30 Jun 1966
2 439 671.5	86 401	6	# 30 Jun 1967
2 440 037.5	86 401	7	# 30 Jun 1968
2 440 402.5	86 401	8	# 30 Jun 1969
2 440 767.5	86 401	9	# 30 Jun 1970
2 441 132.5	86 401	10	# 30 Jun 1971
2 441 498.5	86 401	11	# 30 Jun 1972
2 441 682.5	86 401	12	# 31 Dec 1972
2 442 047.5	86 401	13	# 31 Dec 1973

Table 2: Extraordinary days from 1 Jan 1700 to 2 Jan 2100

Julian Day Number	length in seconds	DTAI	Day Month Year
2 442 412.5	86 401	14	# 31 Dec 1974
2 442 777.5	86 401	15	# 31 Dec 1975
2 443 143.5	86 401	16	# 31 Dec 1976
2 443 508.5	86 401	17	# 31 Dec 1977
2 443 873.5	86 401	18	# 31 Dec 1978
2 444 238.5	86 401	19	# 31 Dec 1979
2 444 785.5	86 401	20	# 30 Jun 1981
2 445 150.5	86 401	21	# 30 Jun 1982
2 445 515.5	86 401	22	# 30 Jun 1983
2 446 246.5	86 401	23	# 30 Jun 1985
2 447 160.5	86 401	24	# 31 Dec 1987
2 447 891.5	86 401	25	# 31 Dec 1989
2 448 256.5	86 401	26	# 31 Dec 1990
2 448 803.5	86 401	27	# 30 Jun 1992
2 449 168.5	86 401	28	# 30 Jun 1993
2 449 533.5	86 401	29	# 30 Jun 1994
2 450 082.5	86 401	30	# 31 Dec 1995
2 450 629.5	86 401	31	# 30 Jun 1997
2 451 178.5	86 401	32	# 31 Dec 1998
2 453 735.5	86 401	33	# 31 Dec 2005
2 454 831.5	86 401	34	# 31 Dec 2008
2 456 108.5	86 401	35	# 30 Jun 2012
2 457 203.5	86 401	36	# 30 Jun 2015
2 457 753.5	86 401	37	# 31 Dec 2016
2 462 317.5	86 399	36	# 30 Jun 2029
2 467 249.5	86 401	37	# 31 Dec 2042
2 468 891.5	86 401	38	# 30 Jun 2047
2 470 536.5	86 401	39	# 31 Dec 2051
2 472 179.5	86 401	40	# 30 Jun 2056
2 473 640.5	86 401	41	# 30 Jun 2060
2 475 101.5	86 401	42	# 30 Jun 2064

Table 2: Extraordinary days from 1 Jan 1700 to 2 Jan 2100

Julian Day Number	length in seconds	DTAI	Day Month Year
2 476 380.5	86 401	43	# 31 Dec 2067
2 477 657.5	86 401	44	# 30 Jun 2071
2 478 937.5	86 401	45	# 31 Dec 2074
2 480 033.5	86 401	46	# 31 Dec 2077
2 481 129.5	86 401	47	# 31 Dec 2080
2 482 224.5	86 401	48	# 31 Dec 2083
2 483 320.5	86 401	49	# 31 Dec 2086
2 484 416.5	86 401	50	# 31 Dec 2089
2 485 328.5	86 401	51	# 30 Jun 2092
2 486 242.5	86 401	52	# 31 Dec 2094
2 487 154.5	86 401	53	# 30 Jun 2097
2 488 068.5	86 401	54	# 31 Dec 2099

Another way of presenting the change in DTAI over time is by a series of charts. Figure 1 shows DTAI from the year −2000 to 2500, 2 from 1500 to 2050, 3 from 1600 to 2050, 4 from 1700 to 2050, 5 from 1800 to 2050, 6 from 1900 to 2050, 7 from 1950 to 2050 and 8 from 2000 to 2100.

The first chart shows only a smooth line. Subsequent charts focus closer to the present and so show the randomness in the Earth's rate of rotation that has been measurable since the invention of the telescope and the pendulum clock.

8 POSIX

POSIX is a portable operating system interface based on Unix, intended to maintain compatibility between operating systems.[6] The way POSIX represents time has been criticized for not handling UTC well.[7,8] Markus Kuhn[9] and David Madore[10] have made proposals to improve the way POSIX deals with extraordinary days. These proposals require a table of extraordinary days. Implementations of these proposals which wish to deal with dates before 1972 can use table 2.

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[6] https://pubs.opengroup.org/onlinepubs/9699919799/

[7] https://stackoverflow.com/questions/16539436/unix-time-and-leap-seconds

[8] https://news.ycombinator.com/item?id=4112566

[9] https://www.cl.cam.ac.uk/ mgk25/time/c/

[10] http://www.madore.org/ david/computers/unix-leap-seconds.html

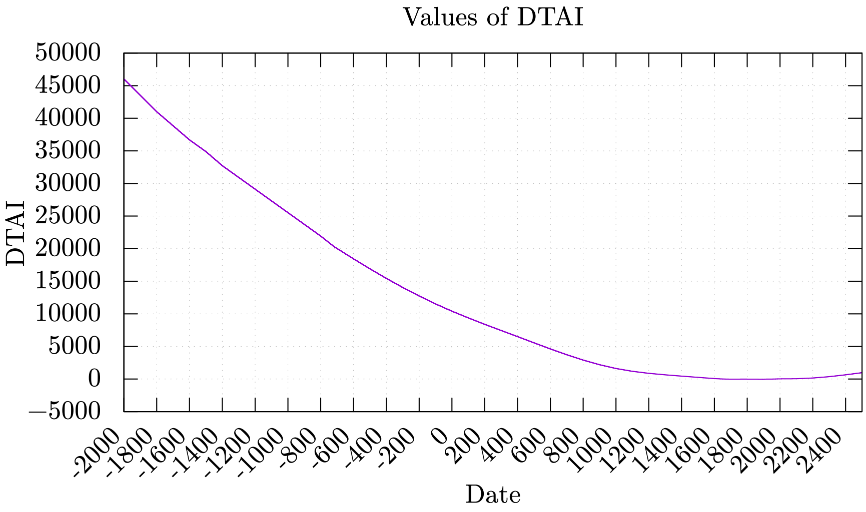

Figure 1: Values of DTAI from −2000 to 2500

9 Conclusion

The exact choice of when to have days with 86 399 or 86 401 seconds in ancient times is somewhat arbitrary, since we don't have sufficiently accurate records of the Earth's rotation rate before the invention of the telescope and the pendulum clock. Indeed, prior to the invention of atomic chronometers in 1955 there were no practically available clocks which were as stable as the rotation of the Earth. Nevertheless, this time scale can be useful for translating ancient times more precise than days into modern terminology for the general reader.

I propose to name this time scale "proleptic UTC with leap seconds".

10 Acknowledgments

I am grateful to the participants of the leapseconds[11] mailing list for their comments on a previous draft of this paper. Zefram,[12] in particular, provided an extensive and detailed review, which improved the paper considerably.

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[11] https://pairlist6.pair.net/mailman/listinfo/leapsecs

[12] See more work by Zefram at https://www.fysh.org/ zefram/, especially Programming Perspective on Time Scales.

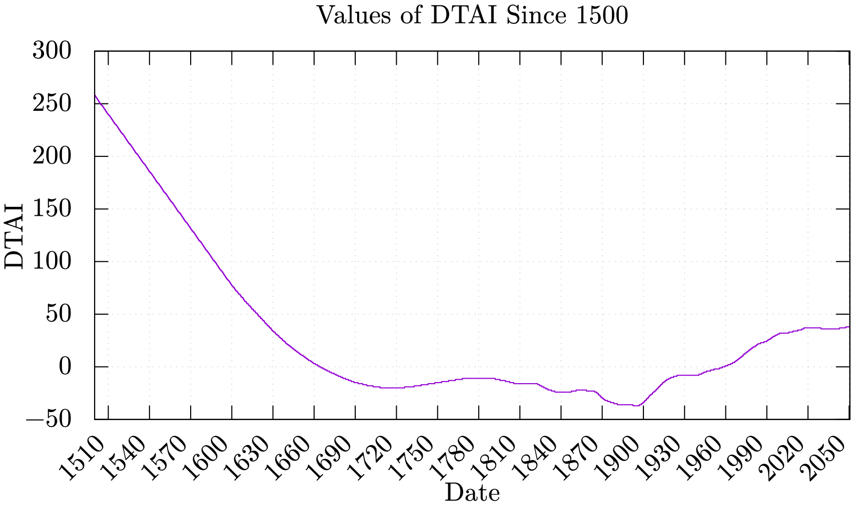

Figure 2: Values of DTAI since 1500

11 Further Reading

Many good research papers are hidden behind “paywalls” and hence are not accessible to an unaffilited reader of modest means. Where I have been able to locate a downloadable copy of these articles, the References section contains the URL.

- F. Richard Stephenson, *Historical Eclipses and Earth’s Rotation: 700 BC–AD 1600* [Stephenson, 2011]

  Abstract: For the whole of the pre-telescopic period, eclipse observations have proved to be by far the best data with which to determine changes in the Earth’s rate of rotation. These changes – on the scale of milliseconds – are produced by both the tides and a variety of non-tidal mechanisms. Each individual observation leads to a result for ΔT (the cumulative effect of changes in the Earth’s spin rate). Over a period of many centuries, this parameter can attain several hours and thus can be determined using fairly crude observations.

  Recently I have extended previous investigations by introducing hitherto unused observations and reinterpreting some of the more reliable existing data: especially in the periods from 700 BC to 50 BC and from AD 300 to 800. This has led to the derivation of revised ΔT values over much of the historical period.

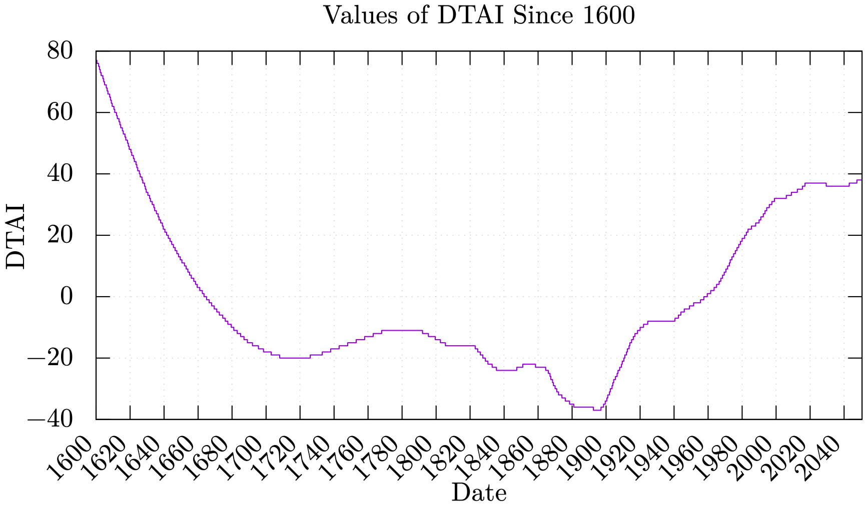

Figure 3: Values of DTAI since 1600

- D. D. McCarthy and A. K. Babcock, *The length of day since 1656* [McCarthy and Babcock, 1986]

  Abstract: Observed values of the difference between the time determined using the rotation of the Earth and a uniform time scale are available since 1627, with useful observations becoming available in 1656. These early data were recorded with low precision and must be smoothed numerically in order to be useful for the derivation of estimates of the difference between the actual length of the day and the standard 24 h, known as excess length of day, or the Earth's rotational speed. Modern data requires no smoothing. The observational data have been adjusted to be consistent with one estimate of the lunar acceleration and smoothed so that the internal and external precision of the observations are approximately equal. The final values along with the derived excess length of day are presented and their spectra are discussed.

- C. Jordi, L. V. Morrison, R. D. Rosen, D. A. Salstein and G. Rosseló, *Fluctuations in the Earth's rotation since 1830 from high-resolution astronomical data* [Jordi et al., 1994]

  Summary: Fluctuations in the Earth's rotation since 1830, as evidenced by changes in the length of the day, are derived from astronomical data having subannual resolution. Before 1955.5, timings of lunar occultations

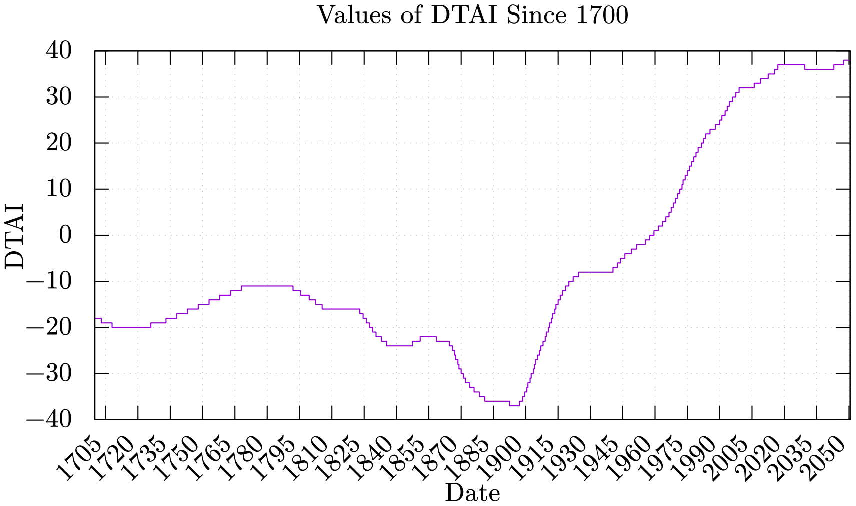

Figure 4: Values of DTAI since 1700

are used; after 1955.5, the data are taken from the time series TAI-UT1.

Although the data in the earliest period, 1830–90, display decade fluctuations in the length of the day, they are not accurate enough to reveal interannual variations. In this regard, also, the results from 1890–1925 are somewhat dubious. The quality of the data after 1925, though, is such that the temporal behaviour of the interannual fluctuations in the length of the day can be traced with confidence. We present plots of the interannual fluctuations in the period 1890–1987 and the longer-term decade fluctuations in the period 1830–1983.

The interannual fluctuations in the length of the day since 1925 are compared with an index of the El Niño/Southern Oscillation (ENSO) phenomenon in the ocean-atmosphere system and are subjected to spectral analysis. The results support the conclusions reached by other authors that these fluctuations are linked to circulation changes in the atmosphere associated with ENSO, and in part to the quasi-biennial oscillation in the equatorial stratosphere's zonal winds. A spectral analysis of our 62 yr series of length of day values since 1925 reveals two significant peaks in the interannual range 2–4 yr. One is roughly biennial and the other is about twice this period, broadly supporting results obtained previously from shorter records.

Our analysis of high-resolution data, therefore, contributes to ongoing

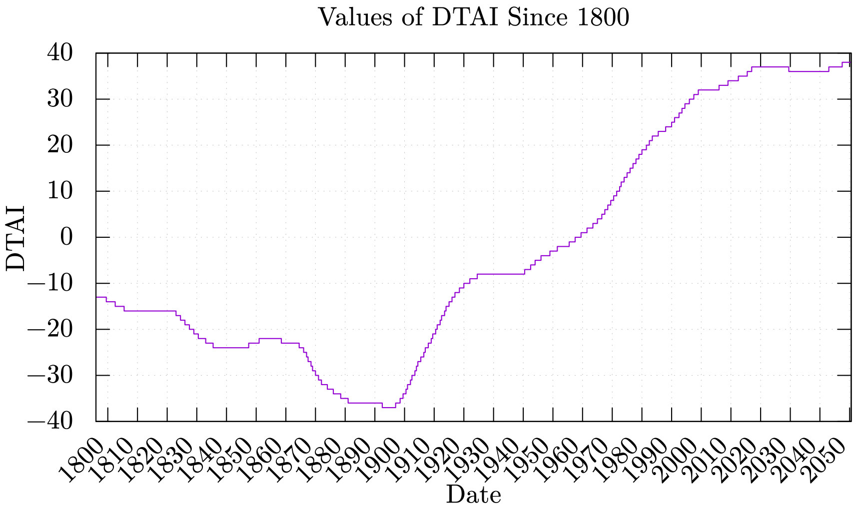

Figure 5: Values of DTAI since 1800

efforts to establish a close relationship between the length of the day and aspects of the global climate system in the period before modern data became available in 1955.5.

- Dennis D. Mccarthy and Brian J. Luzum, *An Analysis of Tidal Variations in the Length of Day* [Mccarthy and Luzum, 1993]

  Summary: Observations of the length of day, corrected for the effects of variations in the angular momentum due to changes in wind velocity and atmospheric pressure, ocean-tide heights and currents, and solid-Earth zonal tides, were analysed. The (1992) IERS Standards model for the effects of zonal tides on the Earth's rotation, which includes ocean-tidal effects, adequately accounts for the observations of the high-frequency (periods between one and 30 days) variations in the length of day at the present level of accuracy. A currently unexplained semi-annual variation in the length of day remains, but this may be due to the unmodelled effects of stratospheric winds. The power spectrum of the remaining variations with periods less than 20 days is essentially that of a white-noise process. The amplitudes of the remaining unexplained variations in length of day are less than 30 microseconds.

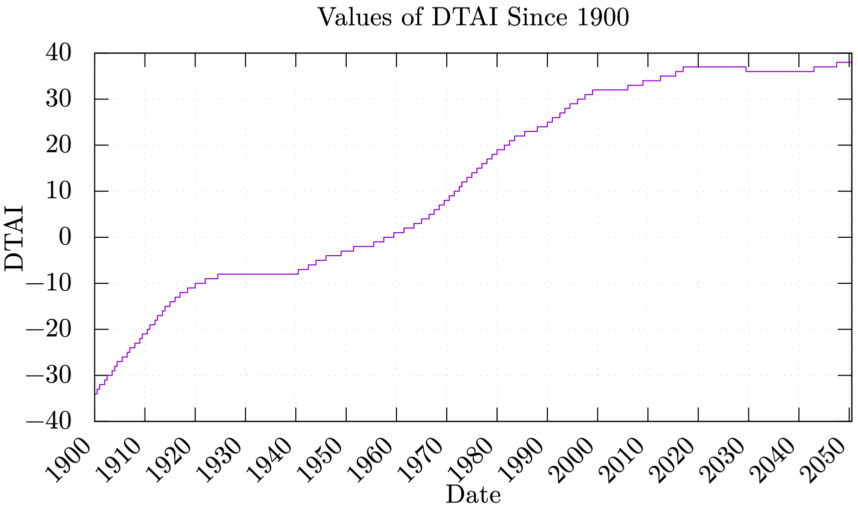

Figure 6: Values of DTAI since 1900

12 Embedded Files

For the convenience of the reader, there are some files embedded in the PDF of this paper. You can extract them using pdfdetach. You can also try to use Adobe Acrobat Reader, but it may refuse to extract some attachments. Except as noted, the embedded files are covered by the same license as this paper.

1. Files `20160404.full.pdf` and `rspa2020776_si_001.pdf`, which are the main research papers on reducing historical records to ΔT, and `R-REC-TF.460-6-200202-I!!PDF-E.pdf`, which is the definition of UTC. They are referenced in this paper by name and with a hyperlink. The files are embedded in this PDF in case the hyperlinks become broken. I do not assert copyright on these files.

2. A spreadsheet named `values_of_delta_T.ods` containing the values of ΔT shown in table 1. Also included is the same data in a CSV file.

3. A plain text file named `legalcode.txt`, downloaded from the Creative Commons web site.[13] In this file is the legal code for the license for this paper, in case the hyperlink in section 13 should fail. I do not assert copyright on this file.

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[13] https://creativecommons.org/licenses/by-sa/4.0/legalcode.txt

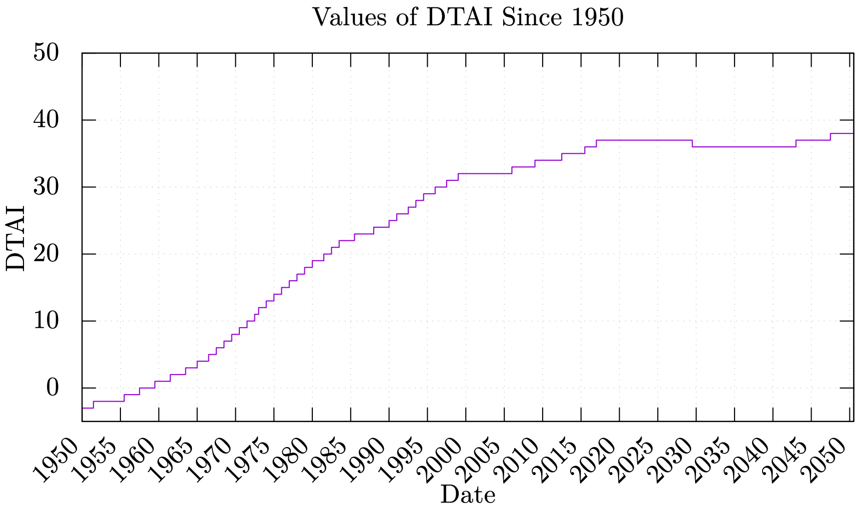

Figure 7: Values of DTAI since 1950

4. A plain text file named `extraordinary_days.dat` which lists the extraordinary days using the form shown in table 2. This file lists all of the extraordinary days plotted in figure 1, not just the subset in table 2.

5. A Python script in file `read_extraordinary_days_table.py` which reads the `extraordinary_days.dat` data file. This program is provided as an example of how to read the data file. It will optionally write the LaTeX code for table 2 and a GNUplot data file for the figures. This program is licensed under the GPL.

6. A plain text copy of the GNU General Public License (GPL), version 3.

7. The source files for this paper. The main file is `proleptic_UTC.tex.in`. It is converted to `proleptic_UTC.tex` which is processed by LuaLaTeX. That file reads LaTeX files `extraordinary_days.tex` and `delta_t.tex`, the `.tex` files for the figures and `proleptic_UTC.bbl` directly, and copies all the embedded files to the output file, `proleptic_UTC.pdf`. You can use these files if you wish to publish a derivative of this paper, perhaps with errors corrected and your own ideas added.

   To reproduce this PDF, extract all the files and run lualatex on `proleptic_UTC.tex`. You will have to run it more than once to get the forward references to resolve.

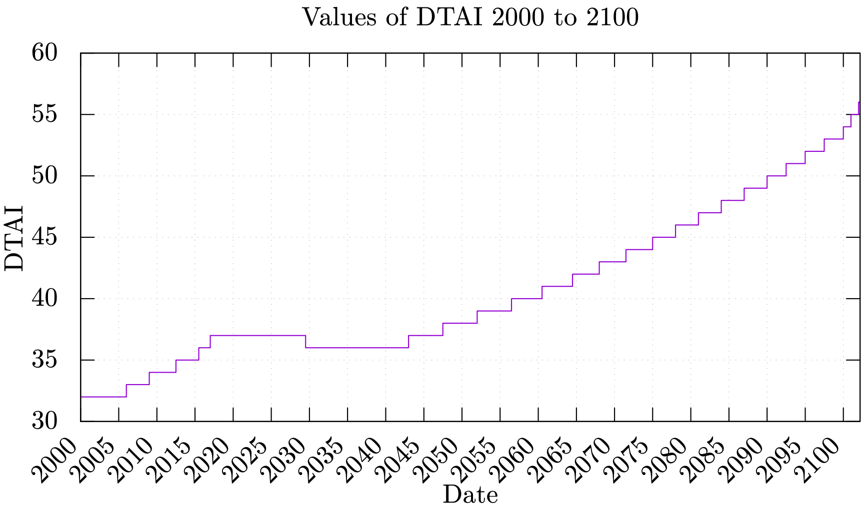

Figure 8: Values of DTAI from 2000 to 2100

8. Also included are files to automate the creation of `extraordinary_days.dat` and this paper when the IERS updates its information about the Earth's rotation. This is also useful if you modify the paper as described above. See file `README` for details. You will need Autotools, Python, LaTeX and GNUplot. Details of the dependencies are in file `proleptic_utc_with_leap_seconds.spec`.

   Update file `references.bib` if you modify, add or remove any of the literature references.

   If you wish to change the algorithm for converting ΔT to extraordinary days, change `read_delta_t.py` to implement your new algorithm and run it to create the data lines for file `extraordinary_days.dat`. See `Makefile.am` for details.

   If you wish to update the values of ΔT, for example to take advantage of new research, if you have LibreOffice you can update `values_of_delta_T.ods` and save a copy as `values_of_delta_T.csv`. If you don't have LibreOffice you can use a text editor to update file `values_of_delta_T.csv` directly. Then re-run `read_delta_t` as above and run `reformat_delta_t` as follows:

   ```
   python3 reformat_delta_t.py values_of_delta_T.csv delta_t.tex
   ```

   File `extraordinary_days.dat` is built from files `exdays_01.dat`, `exdays_02.dat`, `exdays_03.dat`, `exdays_04.dat` and `exdays_05.dat` using the procedure described in `Makefile.am`. Note that file `exdays_01.dat` has the heading comments, including the revision date, which you can edit. File `Makefile.am` also has the recipes for building `gnuplot.dat` and `extraordinary_days.tex`.

To update the figures after changing the schedule of extraordinary days you will need GNUplot. All of the files needed are embedded in this PDF. See file `Makefile.am` for details.

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